

Warm-up!

Solve each of the following

A) $2^x = 16$
 $x = 4$

$$\log_2 16 = 4$$

B) $3^x = \frac{1}{3}$
 $x = -1$

$$\log_3 \left(\frac{1}{3}\right) = -1$$

C) $71^x = 1$

$$\log_{71} 1 = 0$$

D) $25^x = 5$

$$\log_{25} 5 = \frac{1}{2}$$

E) $27^x = 9$

$$\log_{27} 9 = \frac{2}{3}$$

$$27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$$

Log Functions Day 1:

- (2.9) What is a Log?
- (2.12) Log Properties
- (2.9) Log Scales

Homework: Evaluating Logs
and Log Properties HW

Definition of Logarithm

Let b and y be the positive numbers with $b \neq 1$.

$$\log_b y = x \text{ if and only if } b^x = y$$

When you see $\log_b y$, you should read it out loud as "log base b of y "

When you see $\log_b y$, you should think in your head " b to what power makes y ?"

Practice Evaluating

A) $\log_2 64 = 6$

$2^6 = 64$

B) $\log_{169} 13 = \frac{1}{2}$

$169^{1/2} = 13$

C) $\log_6 \left(\frac{1}{36}\right) = -2$

D) $\log_{\frac{1}{3}} 27 = -3$

$\left(\frac{1}{3}\right)^{-3} = \left(3\right)^3 = 27$

* E) $\log_9 \left(\frac{1}{3}\right) = -\frac{1}{2}$ $\rightarrow = \frac{\log_3 \left(\frac{1}{3}\right)}{\log_3 9} = \frac{-1}{2}$

* F) $\log_8 4 = \frac{2}{3}$ $\rightarrow = \frac{\log_2 4}{\log_2 8} = \frac{2}{3}$

G) $\log_{10} 1000 = 3$
 $\log(1000)$

Logs for the Lazy

$$\log_{10} x = \log x \quad \text{Common log}$$

$$\log_e x = \ln(x) \quad \text{Natural log}$$

Two Logs to Know Quickly

$$\log_b 1 = 0$$
$$b^0 = 1$$

$$\log_b b = 1$$

To check yourself when evaluating logs, know how to convert between exponential form and logarithmic form

Rewrite each equation in its equivalent exponential form

A) $\log_5 25 = 2$

B) $\ln 64 \approx 4.159$

Write each expression in its equivalent logarithmic form

C) $1000 = 10^3$

D) $\frac{1}{32} = 16^{-\frac{5}{4}}$

How well do you remember log properties from Algebra II? Four of the following are true. Four are false. Figure out which is each.

A) $\log(3^5) = (\log 3)^5$ F
 $5 \log 3$

B) $\log(12) = \log(3) + \log(9)$ F

C) $\log(27) = \log(3) + \log(9)$ T
 $\log(3 \cdot 9)$

D) $\log(\sqrt{50}) = \frac{1}{2} \log 50$ T
 $\log(50^{\frac{1}{2}}) = \frac{1}{2} \cdot \log 50$

E) $\log(5) = \log(10) - \log(2)$ T
 $\log\left(\frac{10}{2}\right)$

F) $\log(3) = \frac{\log 12}{\log 4}$ F
 $\log\left(\frac{12}{4}\right) = \log 12 - \log 4$

G) $\frac{\log 18}{\log 3} = \log 18 - \log 3$ F
 $\log\left(\frac{18}{3}\right)$

H) $\frac{\log 18}{\log 3} = \log_3 18$ T

$$\frac{\log 18}{\log 3} = \frac{\ln 18}{\ln 3} = \frac{\log_7 18}{\log_7 3} = \frac{\log_5 18}{\log_5 3}$$

The Change of Base Formula

$$\log_b x = \frac{\log_c x}{\log_c b} = \frac{\log_{10} x}{\log_{10} b} = \frac{\ln x}{\ln b}$$

Examples:

$$\log_3 15 = \frac{\log 15}{\log 3} = \frac{\ln 15}{\ln 3}$$

$$\log_{\left(\frac{1}{9}\right)} 27 = \frac{\log_3 27}{\log_3 \left(\frac{1}{9}\right)} = \frac{3}{-2} = -\frac{3}{2}$$

Leveraging Change of Base Formula

$$\text{A) } \log_{16} \left(\frac{1}{32} \right) = \frac{\log_2 \left(\frac{1}{32} \right)}{\log_2 16} = \frac{-5}{4} = -\frac{5}{4}$$

$2^{-5} = \frac{1}{32}$

$$\text{B) } \log_{\sqrt[3]{5}} (\sqrt{5}) = \frac{\log_5 \sqrt{5}}{\log_5 \sqrt[3]{5}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2}$$

$2^5 = 32$

~~C) Write $\log_{\sqrt{2}} 3$ as a ratio of logs that could be used in a calculator that only has common log and natural log~~

Properties of logs

1) Product Rule: $\log_b(xy) = \log_b x + \log_b y$

$$\log(ABC) = \log A + \log B + \log C$$

2) Quotient Rule: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$$\log\left(\frac{AD}{BC}\right) = \log A - \log B - \log C + \log D$$
$$\log A + \log D - \log B - \log C$$

3) Power Rule: $\log_b(x^c) = c \cdot \log_b(x)$

$$\log\left(\frac{AB}{C^3}\right) = \log A + \log B - 3 \log C$$

$$\log\left(\frac{\sqrt{A}}{B^3 C^5}\right) = \frac{1}{2} \log A - 3 \log B - 5 \log C$$

$$\log\left(\frac{1}{A^2 B}\right) = -2 \log A - \log B$$

$$\log\left(\frac{100}{A^2 B}\right) = \log(100) - 2 \log A - \log B$$
$$= 2 - 2 \log A - \log B$$

More Practice Evaluating

$$\log_2 16$$

$$\log \left(\frac{1}{1000} \right)$$

$$\ln \left(\frac{1}{e} \right)$$

$$\log_6 \sqrt[3]{36}$$

$$\log_4 2$$

$$\log_{32} \left(\frac{1}{2} \right)$$

$$\log_3 \left(\frac{1}{81} \right)$$

$$\log \left(\sqrt{\frac{1}{10}} \right)$$

Rewrite by condensing into a single logarithm

$$\text{A) } \log_2 5 + 2 \log_2 x - \log_2 3 - \log_2 y = \log_2 \left(\frac{5x^2}{3y} \right)$$

$$\text{B) } \ln(x) + \frac{1}{2} \ln(x+2) = \ln \left(x (x+2)^{\frac{1}{2}} \right)$$

Rewrite by condensing into a single logarithm

$$c) 6 \ln(x+1) - 2 \ln x - \ln(x^2-2) = \ln \left(\frac{(x+1)^6}{x^2(x^2-2)} \right) = \ln \left(\frac{(x+1)^6}{x^4-2x^2} \right)$$

$$D) 2 \log_5 x - \frac{1}{2} \log_5 (x-1) = \log_5 \left(\frac{5^2}{x(x-1)^{1/2}} \right) = \log_5 \left(\frac{25}{x\sqrt{x-1}} \right)$$
$$2 \log_5 5 - \log_5 x - \frac{1}{2} \log_5 (x-1)$$

Rewrite by condensing into a single logarithm

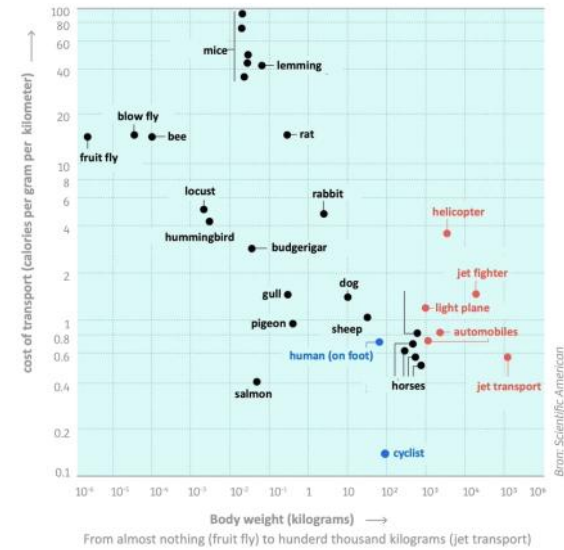
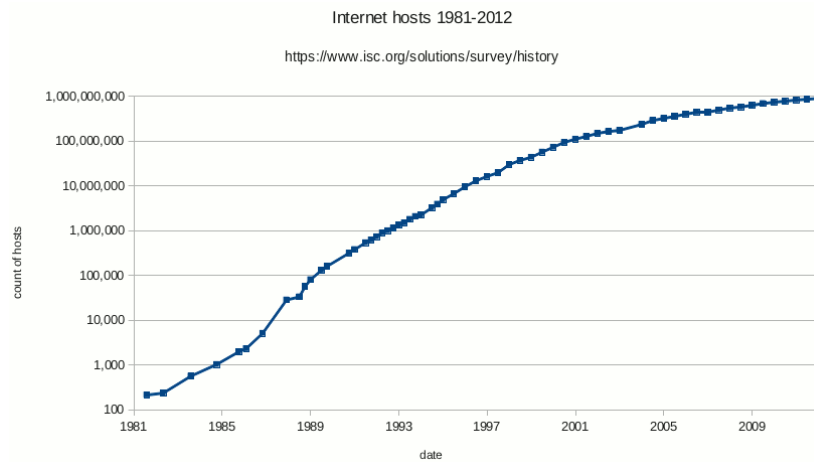
E) $-2 - \ln 4$

$$-2 \ln(e) - \ln 4 = \ln\left(\frac{1}{4e^2}\right)$$

F) $1 + \log x - \log(x^2 - 1) = \log\left(\frac{10x}{x^2 - 1}\right)$

$$1 \log_{10} 10 + \log_{10} x - \log_{10}(x^2 - 1) = \log_{10}\left(\frac{10^1 x}{(x^2 - 1)}\right)$$

Sometimes, for quantities that vary by orders of magnitude, instead of using a "linear" scale, we use a "logarithmic" scale.



When it makes sense to compare things with a proportion (how much to multiply by) rather than a difference (how much to add), a logarithmic scale is a good idea.

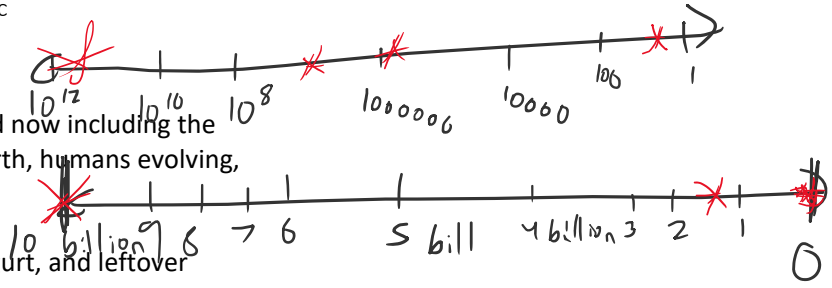
Which of the following may make sense to plot using a logarithmic scale?

A timeline of "important events" between the big bang and now including the formation of the solar system, the emergence of life on earth, humans evolving, and the iPhone being invented.

The cost of things in your refrigerator like orange juice, yogurt, and leftover pasta.

The number of pages in various books in the library such as "Lord of the Flies," "Catch-22," "Jane Eyre," and "Beloved."

The distance between Cypress Bay and fun places like Islands of Adventure, Disneyland Paris, Mars, Proxima Centauri (the nearest star other than the sun)



linear

linear